Bigly Wiggly

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6 May 2017

This is about how to answer the question, "How do we define the **size** of an object quantum mechanically?" The cutesie-pie title is inspired by the current POTUS and the fact that in quantum mechanics (QM) *every object is both a particle and a wave*. (Waves "wiggle", get it? And who can resist poking fun at Trump?)

For objects as big as baseballs, or even BBs, their *wavelength* is such a tiny fraction of their macroscpoic physical dimensions that their wave nature is unobservable, so the question sounds pretty silly. The size (diameter) of an official baseball is 72.6474.68 mm. Most BBs are 4.5 mm (0.177 in) in diameter.

What about a grain of sand? *Wikipedia* says sand grains span the range from 2 mm down to 0.004 mm in diameter. Let's take the latter size and estimate the mass, assuming it is made of quartz, which has a density of 2.65 gm/cm^3: skipping all the arithmetic (trust me!) we get $m = 0.888 \times 10^{-10}$ grams or 0.888×10^{-13} kg.

Now, Louis-Victor-Pierre-Raymond, 7e duc de Broglie, proposed in 1924 that the *wavelength* λ of an object is inversely proportional to its *momentum* p, with a constant of proportionality $h = 6.62607004 \times 10^{-34}$ m^{2}kg/s (Planck's constant). That is,

$$\lambda = h/p \; .$$

Amazingly, this relationship has been confirmed in every situation since then, even for highly relativistic particles!

So it should be easy to calculate the wavelength of the smallest grain of sand, right? Not so fast. We need the *momentum* of the grain, not just its mass. Assuming its velocity is much less than the speed of light, its momentum is the product of its mass and its velocity.

So the *faster* the grain moves, the *shorter* its wavelength! That's weird. Let's take the smallest plausible velocity, namely the average speed it would have if suspended in water at room temperature. Since it would be getting bumped constantly by water molecules with a thermal momentum of about 1.58×10^{-23} kg m/s, it would have a comparable momentum, probably somewhat less than that, so let's assume $p = 10^{-23}$ kg m/s. Plugging that into de Broglie's equation gives $\lambda = 0.66 \times 10^{-10}$ m. The wavelength is still about 60,000 times smaller than the diameter of the smallest grain of sand.

So the "size" of the smallest things we can see with the naked eye is still defined quite consistently by their physical dimensions. No wonder we tend to think of that as an absolute definition! But it isn't. Not if we go still smaller.

I'll skip over all the "microscopic" stuff like blood cells and protein molecules where

things start looking fuzzy, and go straight to the size of a nucleus of carbon, which has a mass of about 2×10^{-26} kg. At room temperature its thermal motion should give it a momentum of about 1.28×10^{-23} kg m/s and therefore a thermal de Broglie wavelength of about 0.5×10^{-10} m. Still pretty small... but how does it compare with the physical diameter of a carbon nucleus, 0.54×10^{-14} m? The wavelength is nearly 10,000 times *larger* than the diameter.

So which do we consider the "size" of a carbon nucleus? At this point the consensus is that the physical diameter of the nucleus is still its "size", since we can *measure* it by shooting electrons at a bunch of such nuclei and seeing what fraction of them hit anything.

Also, of course, we can change the wavelength of the carbon nucleus by increasing its speed: at about six million m/s its wavelength will equal its diameter.

So let's get to the hard part: how do we define "size" for "point particles" that don't seem to *have* any diameter (as far as we have been able to tell by shooting other particles at them)? This is the situation for all the "fundamental" particles listed in the "Standard Model" of elementary particle physics.

Which is more "bigly", an electron or a muon? And how do we decide?

Well, a popular choice is to switch to *mass*. The muon is 207 times more massive than the electron, so it stands to reason that it must be "bigger", right?

Not so much. If both have zero diameter, the only distance scale we have to play with is the wavelength, and that is ambiguous on account of its dependence on the particle's velocity.

But there is another wavelength that is a tempting candidate for the definition of "size": the **Compton wavelength**, $\lambda_C = h/mc$, in which we just use the speed of light for the velocity (ignoring the fact that a particle with mass can never actually reach the speed of light).

This seems like a handy criterion, but it underlines the somewhat counterintuitive fact that *heavier* (more massive) particles are *smaller* than *lighter* ones.

This situation is, I think, an inescapable confounding of our "common sense" about *what can be composed of what*.

I would love to see an audiovisual representation of this breakdown, perhaps substituting darkness and deep tones for "heaviness" and bright colors and piccolo music for the least massive particles, while depicting "wave packets" and "quantum fuzziness" in some form of defocusing or blurring.

Any takers? `:-)`